Hamiltonian analysis for Lifshitz type Fields

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Abstract

Using the Dirac Method, we study the Hamiltonian consistency for three field theories. First we study the electrodynamics a la Hořava and we show that this system is consistent for an arbitrary dynamical exponent z. Second, we study a Lifshitz type electrodynamics, which was proposed in [1]. For this last system we found that the canonical momentum and the electrical field are related through a Proca type Green function, however this system is consistent. In addition, we show that the anisotropic Yang-Mills theory with dynamical exponent $z = 2$ is consistent. Finally, we study a generalized anisotropic Yang-Mills theory and we show that this last system is consistent too.

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1 Introduction

Lately, systems invariant under anisotropic scaling

$$\vec{x} \to b\vec{x}, \qquad t \to b^z t, \quad b = \text{constant}, \tag{1}$$

where z is a dynamical exponent, have been attracted a lot of attention. For example, in general relativity it has been found space-times invariant under the anisotropic scaling (1), see [2]. Some of these kind of space-times can be seen as generalized Schrödinger space-time [3] and other as an AdS deformation [4, 5]. It is worth to mention that different anisotropic space-times are solution for the Einstein's equation with energy momentum tensor produced by a Proca Field [2], which is a massive field. Now, originally the anisotropic scaling (1) were found in condensed matter [6]. Amazingly, the gravity/gauge correspondence allows a relation between different metrics invariant under the anisotropic scaling (1) and some condensed matter systems [7, 8, 9]. Furthermore, almost every field theory can be transformed into a field theory invariant under the anisotropic scaling (1), this deformed field theory improves its high energy behavior [10, 11, 12, 13, 14, 15, 16]. In fact, using the anisotropic scalings (1), Hořava formulated a modified gravity which seems to be free ghosts and power counting renormalizable for $z = 3$ [17]. However, applying Dirac Method, it was shown that this gravity has dynamical inconsistencies [19], but with the same method were found healthy extensions [20, 21]. Hořava gravity has different interesting properties, some works about these properties can be seen in [22, 23, 24, 25, 26, 27, 28, 29]. The Dirac Method is important to understand the anisotropic gravity, however there is not a study about Hamiltonian consistency for anisotropic gauge fields as anisotropic electrodynamics or anisotropic Yang-Mills field.

In this paper, using the Dirac Method, we show that the anisotropic electrodynamics is consistent for an arbitrary dynamical exponent z. In addition, we show that the usual Coulomb gauge condition is correct for this system. Furthermore, we study the Hamiltonian formalism for a Lifshitz type electrodynamics. For this last system we found that the canonical momentum and the electrical field are related through a Proca type Green function, however this system has two degrees of freedom and is consistent. It is worth mentioning that this last system was proposed for generating neutrino masses dynamically [1]. Moreover, using the Dirac Method again, we find that the Hamiltonian formalism for the anisotropic Yang-Mills theory with

$z = 2$ proposed in [16] is consistent. Also, we study a generalized anisotropic Yang-Mills theory and we show that this last system is consistent too.

This paper is organized as follow: in the section 2 we study the formalism for the electrodynamics a la Hořava; in the section 3 we study a Lifshitz type electrodynamics; in the section 4 the anisotropic Yang-Mills is studied and in the section 5 our summary is given.

2 Anisotropic Electrodynamics and Hamiltonian analysis

In this section we study the Hamiltonian formalism for the anisotropic electrodynamics, the action for this system is given by [12]

$$S = \int c dt d\vec{x} \mathcal{L} = \int dt d\vec{x} \left(E_i E_i - \frac{1}{2} F_{ij} f\left(\nabla^2\right) F_{ij} \right), \tag{2}$$

where $f(x) = \sum_{z \geq 1} a_z x^{z-1}$ and

$$E_i = -\left(\partial_t A_i + \partial_i \phi\right), \quad B_i = \left(\vec{\nabla} \times \vec{A}\right)_i, \quad F_{ij} = \partial_i A_j - \partial_j A_i = \epsilon_{ijk} B_k. \tag{3}$$

The case $z = 2$ was first studied in [30].

Now, from the action (2) we obtain

$$\pi^i = \frac{\partial \mathcal{L}}{\partial(\partial_t A_i)} = -2\alpha E_i, \tag{4}$$

$$\pi^0 = \frac{\partial \mathcal{L}}{\partial(\partial_t \phi)} = 0. \tag{5}$$

Then we have the primary constraint

$$\chi_1 = \pi^0(\vec{x}, t) \approx 0. \tag{6}$$

In addition, the canonical Hamiltonian for this system is given by

$$\begin{aligned} H_c(t) &= \int d\vec{x} \left(\pi^i \partial_t A_i - \mathcal{L}\right) \\ &= \int d\vec{x} \left(\frac{1}{4}\pi_i \pi_i + B_i f(\nabla^2) B_i - \phi \partial_i \pi^i\right). \end{aligned} \tag{7}$$

According to Dirac Method [31], all Hamiltonian constraint does not evolve. Then, the constraint (6) must satisfy

$$\dot{\chi}_1 \approx 0, \tag{8}$$

namely

$$\begin{aligned}\dot{\chi}_1 &= \Big\{\pi^0(\vec{x},t), H_c(t)\Big\} = \left\{\pi^0(\vec{x},t), \int d\vec{y}\left(\frac{1}{4}\pi_i\pi_i + B_i f(\nabla^2)B_i - A_0\partial_i\pi^i\right)\right\} \\ &= \partial_i\pi^i(\vec{x},t). \end{aligned}\tag{9}$$

This equation implies the new constraint

$$\chi_2 = \partial_i\pi^i(\vec{x},t) \approx 0. \tag{10}$$

According to Dirac Method [31], this last constraint must satisfy

$$\dot{\chi}_2 \approx 0, \tag{11}$$

namely

$$\begin{aligned}\dot{\chi}_2 &= \Big\{\partial_i\pi^i(\vec{x},t), H_c\Big\} = \left\{\partial_i\pi^i(\vec{x},t), \int d\vec{y}\Big(B_j f(\nabla^2)B_j\Big)\right\} \\ &= \partial_i\epsilon_{ilj}\partial_l\Big(f(\nabla^2)B_j\Big) = 0. \end{aligned}\tag{12}$$

Thus, there are not more constraints. In conclusion, the Hamiltonian constraints are given by

$$\chi_1 = \pi^0(\vec{x},t) \approx 0, \qquad \chi_2 = \partial_i\pi^i(\vec{x},t) \approx 0, \tag{13}$$

which satisfy

$$\{\chi_1(\vec{x},t), \chi_2(\vec{x}',t)\} = 0. \tag{14}$$

For this reason, the constraints (13) are first class constraints and them generate gauge transformations for the system (2). Then, the gauge freedom for this system is the same of the usual electrodynamics. In particular, this system has two degrees of freedom and there are not dynamical inconsistencies.

Furthermore, the extended Hamiltonian is given by

$$H_E(t) = \int d\vec{x}\left(\frac{1}{4}\pi_i\pi_i + B_i f(\nabla^2)B_i - A_0\partial_i\pi^i + \lambda_1\pi^0 + \lambda_2\partial_i\pi^i\right),$$

where λ_1 and λ_2 are Lagrange multipliers. Using this Hamiltonian we have the equations of motion

$$\begin{aligned} \dot{\phi}(\vec{x},t) &= \{\phi(\vec{x},t), H_E(t)\} = \lambda_1(\vec{x},t), & (15) \\ \dot{A}_i(\vec{x},t) &= \{A_i(\vec{x},t), H_E(t)\} = \frac{1}{2\alpha}\pi_i(\vec{x},t) + \partial_i\left[\phi(\vec{x},t) - \lambda_2(\vec{x},t)\right], & (16) \\ \dot{\pi}^0 &= \left\{\pi^0(\vec{x},t), H_E(t)\right\} = \partial_i \pi^i(\vec{x},t) = 0, & (17) \\ \dot{\pi}_i &= \{\pi_i(\vec{x},t), H_E(t)\} = \epsilon_{ilj}\partial_l\left(f(\nabla^2)B_j(\vec{x},t)\right). & (18) \end{aligned}$$

The two last equations can be written as

$$\begin{aligned} \nabla \cdot \vec{E} &= 0, & (19) \\ \vec{\nabla} \times (f(\nabla^2)\vec{B}) &= \frac{\partial \vec{E}}{\partial t}, & (20) \end{aligned}$$

which are the equations of motion for anisotropic electrodynamics [12].

3 Coulomb gauge

Due that this theory has two first class constraints, we have to choice two gauge conditions. The usual Coulomb gauge conditions are

$$\begin{aligned} \chi_3(\vec{x},t) &= \phi(\vec{x},t) \approx 0, & (21) \\ \chi_4(\vec{x},t) &= \partial_i A^i(\vec{x},t) \approx 0, & (22) \end{aligned}$$

which are good gauge conditions for the anisotropic electrodynamics. In fact, according to Dirac Method, the constraints (21)-(22) have not evolve. Then,

$$\begin{aligned} \dot{\phi}(\vec{x},t) &= \{\phi(\vec{x},t), H_E(t)\} = \lambda_1(\vec{x},t) \approx 0, & (23) \\ \dot{\chi}_4 &= \{\partial_i A_i, H_E\} = \left\{\partial_i A_i, H_c + \int d\vec{y}\left(\lambda_1\pi^0 + \lambda_2\partial_j\pi^j\right)\right\} \\ &= \frac{1}{2\alpha}\partial_i\pi_i(\vec{x}) + \nabla^2\phi - \partial_i\partial_i\lambda_2 \approx 0, \end{aligned}$$

which implies

$$\nabla^2\phi - \partial_i\partial_i\lambda_2 \approx 0. \qquad (24)$$

Now, the Dirac bracket for this gauge is

$$\{V(\vec{x},t),W(\vec{y},t)\}^{*} = \{V(\vec{x},t),W(\vec{y},t)\}$$
$$+\int d\vec{y}' \left[\left\{V,\pi^{0}(\vec{y}',t)\right\}\left\{A^{0}(\vec{y}',t),W\right\} - \left\{V,A^{0}(\vec{y}',t)\right\}\left\{\pi^{0}(\vec{y}',t),W\right\}\right]$$
$$-\int\int \frac{1}{4\pi}\frac{1}{\mid \vec{w}-\vec{y}' \mid}\left\{V,\partial_{i}\pi^{i}(\vec{w},t)\right\}\left\{\partial_{i}A^{j}(\vec{y}',t),W\right\} d\vec{y}' d\vec{w}$$
$$+\int\int \frac{1}{4\pi}\frac{1}{\mid \vec{w}-\vec{y}' \mid}\left\{V,\partial_{j}A^{j}(\vec{w},t)\right\}\left\{\partial_{i}\pi^{i}(\vec{y}',t),W\right\} d\vec{y}' d\vec{w}.$$

In particular, we have

$$\begin{aligned} \{A^{\mu}(\vec{x},t),\pi^{\nu}(\vec{y},t)\}^{*} &= \left(\eta^{\mu\nu}+\eta^{\mu 0}\eta^{0\nu}\right)\delta^{3}(\vec{x}-\vec{y}) \\ &- \frac{\partial}{\partial x_{\mu}}\frac{\partial}{\partial y_{\nu}}\left(\frac{1}{4\pi}\frac{1}{\mid \vec{x}-\vec{y} \mid}\right). \end{aligned} \tag{25}$$

Then, the equations of motion are

$$\left\{A^{i}(\vec{x},t),H_{E}(t)\right\}^{*} = \dot{A}^{i}, \tag{26}$$

$$\left\{\pi^{i}(\vec{x},t),H_{E}(t)\right\}^{*} = \vec{\nabla}\times\left(f(\nabla^{2})\vec{B}\right) = \frac{\partial \vec{E}}{\partial t}. \tag{27}$$

Thus, the usual Coulomb gauge is a good gauge condition for the system (2).

4 Lifshitz type electrodynamics

Recently it was proposed a new mechanism to obtain massive fields. In this mechanism spatial higher order derivatives are introduced [1]. For example, the action for the Lifshitz type electrodynamics is

$$S = \int d\vec{x}dt\mathcal{L} = \int d\vec{x}dt\left(-\frac{1}{4}F_{\mu\nu}\left(1-\frac{\nabla^{2}}{M^{2}}\right)F^{\mu\nu}\right), \tag{28}$$

$$= \int d\vec{x}dt\left(\frac{1}{2}E_{i}E^{i} - E_{i}\frac{\nabla^{2}}{2M^{2}}E^{i} - \frac{1}{4}F_{ij}F_{ij} + F_{ij}\frac{\nabla^{2}}{4M^{2}}F_{ij}\right). \tag{29}$$

From this action we have

$$\pi^{0} = 0, \tag{30}$$

$$\pi^{i} = -\left(1-\frac{\nabla^{2}}{M^{2}}\right)E^{i}. \tag{31}$$

Then, we have the primary constraint

$$\chi_1 = \pi^0(\vec{x}, t) \approx 0. \tag{32}$$

In addition, from the equation (31), we have

$$E^i(\vec{x}, t) = \int d\vec{x}' G(\vec{x} - \vec{x}')\, \pi^i(\vec{x}', t), \tag{33}$$

where

$$-\left(1 - \frac{\nabla^2}{M^2}\right) G(\vec{x} - \vec{x}') = \delta^3(\vec{x} - \vec{x}'), \tag{34}$$

namely

$$G(\vec{x} - \vec{x}') = -\frac{M^2}{4\pi} \frac{e^{-M|\vec{x} - \vec{x}'|}}{|\vec{x} - \vec{x}'|}. \tag{35}$$

Notice that this Green function is a Proca type propagator.

Now, the canonical Hamiltonian for this system is given by

$$\begin{aligned} H_c(t) &= \int d\vec{x}(\pi^i \partial_t A_i - \mathcal{L}) \\ &= \int d\vec{x} \left(-\frac{1}{2}\pi^i E_i + \frac{1}{4} F_{ij}F^{ij} - \frac{1}{4M^2} F^{ij} \Delta F_{ij} + \phi \partial_i \pi^i\right) \\ &= \int d\vec{x} d\vec{x}' \left(-\frac{1}{2}\pi^i(\vec{x}, t) G(\vec{x} - \vec{x}')\, \pi_i(\vec{x}', t)\right) \\ &\quad + \int d\vec{x} \left(\frac{1}{4} F_{ij}F^{ij} + \frac{1}{4M^2} F^{ij} \Delta F_{ij} + \phi \partial_i \pi^i\right). \end{aligned} \tag{36}$$

In addition, according to the Dirac Method [31], the constraint (32) has to satisfy

$$\dot{\chi}_1 \approx 0, \tag{37}$$

namely

$$\dot{\chi}_1 = \left\{\pi^0(\vec{x}, t), H_c(t)\right\} = \partial_i \pi^i(\vec{x}, t) \approx 0. \tag{38}$$

This equation implies the new constraint

$$\chi_2 = \partial_i \pi^i(\vec{x}, t) \approx 0, \tag{39}$$

which implies

$$\dot{\chi}_2 \approx 0. \tag{40}$$

It can be shown that

$$\dot{\chi}_2 \;=\; \Big\{\partial_i \pi^i(\vec{x},t), H_c(t)\Big\} = 0. \tag{41}$$

Thus, there are not more constraints. Then, all the Hamiltonian constraints are given by the constraints (32) and (39) which are first class constraints and generate the gauge transformations for this system. For that reason there are not dynamical inconsistencies.

Now, the usual Proca field is massive and has three degrees of freedom. In the Lifshitz type electrodynamics appears a Proca type propagator (35), however it has two first class constraints. For this last reason, the Lifshitz type electrodynamics has two degrees of freedom, as the usual electrodynamics.

5 Anisotropic Yang-Mills field

The action for the anisotropic Yang-Mills field is given by [16]

$$S \;=\; \frac{1}{4}\int dt d\vec{x}\left(\frac{1}{e^2}(E_{ai}E_{ai}) + \beta(D_i F_{aik} D_j F_{ajk})\right), \tag{42}$$

where

$$D_i F_{ajk} = \partial_i F_{ajk} + ig f_{ab}{}^{c} A_{bi} F_{cjk}. \tag{43}$$

This action is invariant under anisotropic scaling for $z = 2$.

From the action (42) we have

$$\pi^i_a \;=\; -\frac{1}{2e^2}E^i_a, \tag{44}$$

$$\pi^0_a \;=\; 0. \tag{45}$$

Then, we have the constraints

$$\chi_{1a} = \pi^0_a \;\approx\; 0. \tag{46}$$

In addition, the canonical Hamiltonian is

$$H_c(t) = \int d\vec{x} \left(e^2 \pi_a^i \pi_a^i - \frac{\beta}{4} \left(D_i F_{aik} D_j F_{ajk} \right) - A_{a0} \left(\partial_i \pi_a^i + igf_{ac}{}^b A_{ci} \pi_b^i \right) \right). \quad (47)$$

Using this Hamiltonian we get

$$\dot{\chi}_{1a} = \{ \pi_a^0(\vec{x},t), H_c(t) \} = \partial_i \pi_a^i(\vec{x},t). + igf_{ae}{}^b \pi_b^i(\vec{x},t) A_{ei}(\vec{x},t) \quad (48)$$

Now, due that the Dirac Method sets

$$\dot{\chi}_{1a} \approx 0, \quad (49)$$

we have the new constraints

$$\chi_{2a}(\vec{x},t) = \partial_i \pi_a^i(\vec{x},t) + igf_{ae}{}^b \pi_b^i(\vec{x},t) A_{ei}(\vec{x},t) \approx 0, \quad (50)$$

which satisfy

$$\{ \chi_{2a}(\vec{x},t), \chi_{2b}(\vec{y},t) \} = igf_{ab}{}^c \chi_{2c}(\vec{y},t) \approx 0. \quad (51)$$

Using the constraints (50) we obtain

$$H_c(t) = \int d\vec{x} \left(e^2 \pi_a^i \pi_a^i - \frac{\beta}{4} \left(D_i F_{aik} D_j F_{ajk} \right) - A_{a0} \chi_{2a} \right). \quad (52)$$

Furthermore we found that

$$\dot{\chi}_{2a} = \{ \chi_{2a}, H_c \} = -\frac{ig\beta}{2} f_{ad}{}^h \left(D_i F_h^{ik})(D_j F_d^{jk} \right) = 0. \quad (53)$$

Then, there are not more constraints and the only constraints for this system are (46) and (50), which are first class constraints. This last result implies that the extended Hamiltonian is given by

$$H_E(t) = \int d\vec{x} \left(e^2 \pi_a^i \pi_a^i - \frac{\beta}{4} \left(D_i F_{aik} D_j F_{ajk} \right) + \lambda_{1a} \chi_{1a} + \left(\lambda_{2a} - A_{a0} \right) \chi_{2a} \right), \quad (54)$$

where λ_{1a} and λ_{2a} are Lagrange multipliers.

It is worth mentioning that the constraints (46) and (50) are the same constraints for the usual Yang-Mills theory [32]. Now, the consistent Coulomb gauge conditions for the usual Yang-Mills theory are given by [32]

$$\begin{aligned} \partial_i A_a^i(\vec{x},t) &= 0, \\ A_a^0(\vec{x},t) - \frac{1}{4\pi}\int d\vec{y}\mathbf{G}_{ab}\left(\vec{x},\vec{y},A\right)(2e^2) ig f_{bc}{}^d \pi_d^i(\vec{y},t) A_{ci}(\vec{y},t) &\approx 0, \end{aligned}$$

where

$$\left(\delta_{ab}\partial_i\partial^i + ig f_{ac}{}^b A_c^i \partial_i\right)\mathbf{G}_{cb}(\vec{x},\vec{y},A) = -4\pi\delta_{ab}\delta(\vec{x}-\vec{y}). \tag{55}$$

It is possible to show that these gauge conditions are good gauge conditions for the anisotropic Yang-Mills with $z=2$.

5.1 General case

For the case $z=2$, an alternative action for the anisotropic Yang-Mills theory is given by

$$S = \frac{1}{4}\int dt d\vec{x}\left(\frac{1}{e^2}E_{ai}E_{ai} + \beta F_{ajk}D^2 F_{ajk}\right), \quad D^2 F_{ajk} = D_i D_i F_{ajk}. \tag{56}$$

In fact, we can propose the general action

$$S = \frac{1}{4}\int dt d\vec{x}\left(\frac{1}{e^2}E_{ai}E_{ai} + F_{ajk} f\left(D^2\right) F_{ajk})\right), \tag{57}$$

where $f(x) = \sum_{z\geq 1} a_z x^{z-1}$. From this action we have

$$\pi_a^i = -\frac{1}{2e^2}E_a^i, \tag{58}$$

$$\pi_a^0 = 0. \tag{59}$$

Namely, we obtain the constraints

$$\chi_{1a} = \pi_a^0 \approx 0. \tag{60}$$

In addition, the canonical Hamiltonian is

$$H_c(t) = \int d\vec{x}\left(e^2\pi_a^i\pi_a^i - \frac{1}{4}F_{ajk} f\left(D^2\right) F_{ajk} - A_{a0}\left(\partial_i\pi_a^i + ig f_{ac}{}^b A_{ci}\pi_b^i\right)\right). \tag{61}$$

Now, using this Hamiltonian we arrive to

$$\dot{\chi}_{1a} = \left\{ \pi^0_a(\vec{x},t), H_c(t) \right\} = \partial_i \pi^i_a(\vec{x},t) + ig f_{ae}{}^b \pi^i_b(\vec{x},t) A_{ei}(\vec{x},t), \tag{62}$$

which implies the constraints

$$\chi_{2a}(\vec{x},t) = \partial_i \pi^i_a(\vec{x},t) + ig f_{ae}{}^b \pi^i_b(\vec{x},t) A_{ei}(\vec{x},t) \approx 0. \tag{63}$$

Due that the constraints $\chi_{1a}(\vec{x},t)$ and $\chi_{2a}(\vec{x},t)$ satisfy (51), these are first class constraints.

Now, using the Jacobi's identity for the structure constants

$$f_{rs}{}^c f_{ac}{}^b + f_{sa}{}^c f_{rc}{}^b + f_{ar}{}^c f_{sc}{}^b = 0,$$

we arrive to

$$\begin{aligned} & \left\{ D_i \pi^i_a(\vec{x},t), \int d\vec{y} F_{bjk}(\vec{y},t) \left(D^2\right)^z F_{bjk}(\vec{y},t) \right\} \\ = \; & 2(ig)^2 A^j_r(\vec{x},t) A^k_s(\vec{x},t) \left(f_{rs}{}^c f_{ac}{}^b + f_{sa}{}^c f_{rc}{}^b + f_{ar}{}^c f_{sc}{}^b \right) \left(D^2\right)^z F_{bjk}(\vec{x},t) = 0. \end{aligned} \tag{64}$$

This last result implies

$$\dot{\chi}_{2a} = \{\chi_{2a}, H_c\} \approx 0. \tag{65}$$

Then, there are not more constraints and this system is consistent. In this case the extended Hamiltonian is given by

$$H_E(t) = \int d\vec{x} \left(e^2 \pi^i_a \pi^i_a - \frac{1}{4} F_{ajk} f\left(D^2\right) F_{ajk} + \lambda_{1a}\chi_{1a} + \left(\lambda_{2a} - A_{a0}\right)\chi_{2a} \right), \tag{66}$$

where λ_{1a} and λ_{2a} are Lagrange multipliers. Another generalized anisotropic Yang-Mills theory was proposed in [18].

The Hamiltonian constraints for anisotropic gravity are very different from the Hamiltonian constraints for the usual gravity. However, the Hamiltonian constraints obtained for the Lifshitz type fields are not different from the Hamiltonian constraints for usual fields. This result is interesting, because if it is possible to obtain a quantum gravity with anisotropic scaling transformations, the anisotropic fields theories will not have problems, in fact them improve its UV behavior.

6 Summary

In this paper, we studied the dynamical consistency for the electrodynamics a la Hořava and we show that this system is consistent for arbitrary dynamical exponent z. In fact, for this system the constraints are the same that the usual electrodynamics. For this reason, a good gauge condition for the usual electrodynamics is a good gauge condition for the anisotropic electrodynamics. In addition, we study a Lifshitz type electrodynamics, which was proposed in [1]. For this last system we found that the canonical momenta and electrical field are related through a Proca type Green function. Also, we show that this last system is consistent. The anisotropic Yang-Mills field was studied too, in this case we show that the anisotropic Yang-Mills theory with dynamical exponent $z = 2$ proposed in [16] is dynamical consistent. Finally, we studied a generalized anisotropic Yang-Mills theory and it was shown that this system is consistent too.

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