

Lorentz violation, Two times physics and Strings

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Abstract

We study a recently proposed generalization of the relativistic particle by Kostelecký, that includes explicit Lorentz violation. We present an alternative action for this system and we show that this action can be interpreted as a particle in curved space with a metric that depends on the Lagrange multipliers. Furthermore, the following results are established for this model: (i) there exists a limit where this system has more local symmetries than the usual relativistic particle; (ii) in this limit if we restore the Lorentz symmetry we obtain a direct relationship with the two time physics; (iii) also we show that if we intend to restore the Poincaré symmetry we obtain the action of the relativistic bosonic string.

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1 Introduction

In several recent publications have been proposed different systems with explicit Lorentz violation. Since, the Lorentz symmetry is one of the cornerstones of the Laws of Physics, these works have attracted a lot of attention. For example, the Hořava's gravity breaks locally this symmetry, but has as consequence a renormalizable gravity [1]. In Quantum Field Theory there are also several studies that consider this symmetry violation [2], a review on this subject can be found in [3]. An interesting point is that various models that break the Lorentz symmetry, at the level of Quantum Field Theory [4], imply a dispersion relation of the form

$$(P_\mu + a_\mu)^2 + m^2 + b \cdot b \mp 2\sqrt{(P \cdot b + a \cdot b)^2 + m^2 b \cdot b} = 0, \quad (1)$$

with a and b two constant vectors. Clearly this dispersion relation violates the Lorentz symmetry. Recently in Ref. [5] was found a mechanical model that implies the dispersion relation (1). This model allows to understand more deeply the kind of phenomena that are involved in the Lorentz symmetry violation. An interesting result is the relationship between this model and the Finsler geometry [6]. Let us also mention here that recently the Finsler geometry was proposed as an alternative to the Minkowski geometry in the High Energy Physics regime close to the Planck scale [7]. In that sense, independently of the origin of the proposed action of [5], this system has very interesting properties that are worth to study.

The purpose of this work is to study several aspects of the action presented in [5], and show that this system has several interesting properties. In particular, we introduce an alternative action to the proposed by Kostelecký, et al [5]. Furthermore, we establish that this action is equivalent to a particle in a curved space, where the metric depends on two Lagrange multipliers. This result exhibit that Finsler geometry could be studied in this way. We also consider the local symmetries of the system and we show that there exist a limit where the model has more local symmetries that the classical relativistic free particle. As another property, we show that if in this limit we tray to recover the Lorentz symmetry we obtain several generalized Lorentz invariant systems. These systems have interesting local symmetries. For example, we show in Sec. 5 that applying the Dirac's method [8], we obtain the action for the two time physics. One of the hallmarks of the two

time physics is that in only one action are unified different models to the level of particle. Consequently this theory acts as a unification model to the level of point particle [9]. Also, we show that, nevertheless the Lorentz symmetry is recovered we still don't have a restoration of the Poincaré symmetry.

Now, the dispersion relation (1) originally appears from a Field Theory, and the action for the point particle is obtained from a simplification of a given Field Theory. It is worth to notice that the same happens in the case of the action for the two time physics, where it is obtained from a reduction of a Field Theory [10]. In this work to recover the Poincaré invariance, we will take the inverse path, i.e. we will transform our point particle action into a field theory. We will see that if we consider that the coordinates depend on other parameter σ , i.e. $X^M = X^M(\tau, \sigma)$, and instead of b we take $T \frac{\partial X^M}{\partial \sigma}$ the system will be invariant under Lorentz and Poincaré transformations. We also show that in this case the action of the relativistic bosonic string is obtained. We must mention that in the case of the Snyder space [11], Yang proposed an extra dimension in such way to become the Snyder space Poincaré invariant [12]. A similar process will be taken in this work to recover the Lorentz and Poincaré symmetries.

This work is organized in the following way: In Section 2 we perform a canonical analysis of the action proposed by Kostelecký [6]. In section 3 we find an alternative action to such system. In Section 4 we study the case where the perturbation is stronger than the usual term. In Section 5 we recover the Lorentz symmetry and we establish a relationship between this system and the two time physics. For the Section 6, we show that in order to recover the Poincaré symmetry we obtain the relativistic string. Finally we summarize our results in Section 7.

2 Action for the system

The action proposed in Ref. [5] is

$$S = \int d\tau \left(-m \sqrt{-\dot{X} \cdot \dot{X}} - a \cdot \dot{X} \pm \sqrt{(b \cdot \dot{X})^2 - (b \cdot b)(\dot{X} \cdot \dot{X})} \right). \quad (2)$$

where $A \cdot A = A_M A^M = \eta_{MN} A^M A^N$ with $N, M = 0, 1, 2, \dots, D$ and $\text{sig}(\eta_{MN}) = (-1, 1, \dots, 1)$. Furthermore a_M and b_M are constant vectors. Note that this

action is invariant under reparametrizations given by $\frac{dX}{d\tau} = \frac{d\tilde{\tau}}{d\tau} \frac{dX}{d\tilde{\tau}}$.

The canonical momenta of the system are

$$P_M = m \frac{\dot{X}_M}{\sqrt{-\dot{X} \cdot \dot{X}}} - a_M \pm \frac{(b \cdot \dot{X}) b_M - (b \cdot b) \dot{X}_M}{\sqrt{(b \cdot \dot{X})^2 - (b \cdot b) (\dot{X} \cdot \dot{X})}}, \quad (3)$$

from these we obtain the canonical Hamiltonian

$$H_c = P \cdot \dot{X} - L = 0. \quad (4)$$

Also, we see that the following relations are satisfied

$$(P + a) \cdot b = m \frac{\dot{X} \cdot b}{\sqrt{-\dot{X} \cdot \dot{X}}}, \quad (5)$$

$$(P_M + a_M)^2 = -m^2 \pm \frac{2m \sqrt{(b \cdot \dot{X})^2 - (b \cdot b) (\dot{X} \cdot \dot{X})}}{\sqrt{-\dot{X} \cdot \dot{X}}} - b \cdot b, \quad (6)$$

introducing (5) in (6), we found

$$(P_M + a_M)^2 + m^2 + b \cdot b \mp 2\sqrt{(P \cdot b + a \cdot b)^2 + m^2 b \cdot b} = 0. \quad (7)$$

The dispersion relation (7) is consistent with several models developed to test a possible violation of the Lorentz symmetry [4].

Using the Dirac's method [8, 13], the total Hamiltonian is

$$H_T = \lambda \Phi, \quad (8)$$

$$\Phi = \frac{1}{2} \left[(P_M + a_M)^2 + m^2 + b \cdot b \mp 2\sqrt{(P \cdot b + a \cdot b)^2 + m^2 b \cdot b} \right], \quad (9)$$

where λ is Lagrange multiplier. The Hamiltonian action results

$$S = \int d\tau (P \cdot \dot{X} - \lambda \Phi). \quad (10)$$

In the next section we will show that exist an alternative action for this system.

3 Action without square roots

For the relativistic particle and inclusive for the string and membranes a more suitable action for describing their variational principles is to avoid the introduction of the square root by including a Lagrange multiplier. In the case of the point-particle with Lorentz-violation (2) we have two independent square roots so we need to introduce a pair of Lagrange multipliers. The resulting action is given by

$$S = \int d\tau \left\{ \frac{1}{2} \left[\frac{\dot{X}^2}{\lambda} - \lambda m^2 - 2a \cdot \dot{X} \pm \frac{(b \cdot \dot{X})^2 - b^2 (\dot{X} \cdot \dot{X})}{\beta} \pm \beta \right] \right\}. \quad (11)$$

This action is equivalent to (2) if we use the equations of motion of the two Lagrange multipliers. The above action (11) can be rewritten as

$$S = \int d\tau \left\{ \frac{1}{2} [g_{MN} \dot{X}^M \dot{X}^N - 2a \cdot \dot{X} - \lambda m^2 \pm \beta] \right\}, \quad (12)$$

where the metric g_{MN} is a deformation of the standard Minkowski metric given by

$$g_{MN} = \left(\frac{\beta \mp \lambda b^2}{\lambda \beta} \right) \eta_{MN} \pm \frac{b_M b_N}{\beta}. \quad (13)$$

Using the action (12) is easy to see that in the limit $b \rightarrow 0$ we recover the relativistic particle coupled to external electromagnetic field a . To compute the Hamiltonian we use the action (12) and we get the momenta

$$p_M = g_{MN} \dot{X}^N - a_M. \quad (14)$$

Then the canonical Hamiltonian is

$$H_C = \frac{g^{MN}}{2} (P_M + a_M) (P_N + a_N) + \frac{1}{2} (\lambda m^2 \mp \beta), \quad (15)$$

with the inverse metric g^{MN} results

$$g^{MN} = \frac{\lambda \beta}{\beta \mp b^2 \lambda} \eta^{MN} \mp \frac{\lambda^2}{\beta \mp \lambda b^2} b^M b^N. \quad (16)$$

Furthermore we have two primary constraints

$$p_\lambda \approx 0, \quad p_\beta \approx 0, \quad (17)$$

and in consequence the total Hamiltonian is

$$H_T = H_C + \mu_1 p_\lambda + \mu_2 p_\beta. \quad (18)$$

From the evolution of the primary constraints we get

$$\begin{aligned} \dot{p}_\lambda &= \{p_\lambda, H_T\} = -\frac{\partial g^{MN}}{\partial \lambda} \frac{1}{2} (P_M + a_M) (P_N + a_N) - \frac{m^2}{2}, \\ \dot{p}_\beta &= \{p_\beta, H_T\} = -\frac{\partial g^{MN}}{\partial \beta} \frac{1}{2} (P_M + a_M) (P_N + a_N) \pm \frac{1}{2}. \end{aligned}$$

That results in the two following conditions

$$\left(\beta^2 \eta^{MN} + \left(b^2 \lambda^2 \mp 2\lambda\beta \right) b^M b^N \right) (P_M + a_M) (P_N + a_N) + m^2 \left(\beta \mp b^2 \lambda \right)^2 \approx 0, \quad (19)$$

$$\left(\pm b^2 \eta^{MN} \mp b^M b^N \right) (P_M + a_M) (P_N + a_N) \pm \left(\beta \mp b^2 \lambda \right)^2 \approx 0. \quad (20)$$

Solving from (20) for β we get

$$\beta = \lambda \left(\pm b^2 + \sqrt{A} \right), \quad (21)$$

with A given by

$$A = \left(-\eta^{MN} b^2 + b^M b^N \right) (P_M + a_M) (P_N + a_N). \quad (22)$$

It should be noted that we can rewrite the metric in terms of the momenta or the velocities and it is interesting to observe that at this moment it is not of the Finsler type, since we still haven't eliminated the Lagrange multiplier λ . For the metric we obtain,

$$g_{MN} = \frac{1}{\lambda \left(\pm b^2 + \sqrt{A} \right)} \left(\sqrt{A} \eta_{MN} \pm b_M b_N \right). \quad (23)$$

Furthermore, from (19) using β given by (21) we recover the constraint given in (6).

4 Strong perturbation limit

The action (2) has been proposed to study the Lorentz symmetry breaking, where the usual term is greater than the perturbation that breaks the symmetry. But it is interesting to study what happen at the contrary, i.e., if

the correction is bigger than the usual relativistic term. In other words we are considering the ultrarelativistic regime, where we assume that a_M, b_M are such that

$$\left| m\sqrt{-\dot{X} \cdot \dot{X}} \right| \ll \left| -a \cdot \dot{X} \pm \sqrt{(b \cdot \dot{X})^2 - (b \cdot b)(\dot{X} \cdot \dot{X})} \right|, \quad (24)$$

in this regime we obtain the action

$$S = \int d\tau \left(-a \cdot \dot{X} \pm \sqrt{(b \cdot \dot{X})^2 - (b \cdot b)(\dot{X} \cdot \dot{X})} \right). \quad (25)$$

For the canonical momenta we have

$$P_M = a_M \pm \frac{(b \cdot \dot{X}) b_M - (b \cdot b) \dot{X}_M}{\sqrt{(b \cdot \dot{X})^2 - (b \cdot b)(\dot{X} \cdot \dot{X})}}, \quad (26)$$

and now these satisfy two primary constraints

$$\Phi_1 = (P + a) \cdot b = 0, \quad (27)$$

$$\Phi_2 = (P_M + a_M)^2 + b \cdot b = 0. \quad (28)$$

In this way, the total Hamiltonian is

$$H = \lambda_1 \Phi_1 + \lambda_2 \Phi_2. \quad (29)$$

The constraints Φ_1 and Φ_2 are first class, since

$$\{\phi_1, \phi_2\} = 0. \quad (30)$$

This shows that the action (25) has more local symmetries than the original action (2). In this case the gauge symmetries are given by

$$\begin{aligned} \delta_1 X_M &= \epsilon_1(\tau) b_M, & \delta P_M &= 0, & \delta \lambda_1 &= \dot{\epsilon}_1(\tau), \\ \delta_2 X_M &= \epsilon_2(\tau) 2(P_M + a_M), & \delta P_M &= 0, & \delta \lambda_2 &= \dot{\epsilon}_2(\tau). \end{aligned} \quad (31)$$

5 Lorentz symmetry and two time physics

By eliminate the usual term $m\sqrt{-\dot{X} \cdot \dot{X}}$ in the action (2) we have lost the usual relativistic particle. However, we have more local symmetries. Now,

we will see that by recovering the Lorentz symmetry we will get still more local symmetries and we can related this system to the action of two time physics.

By simplicity we consider $a_M = 0$, in this case the Lagrangian of the action (25) takes the form

$$L = \pm \sqrt{(b \cdot \dot{X})^2 - (b \cdot b) (\dot{X} \cdot \dot{X})}. \quad (32)$$

Thus, to restore the Lorentz symmetry we regard now the constant vector b^M as a local field $B^M = B^M(X)$, transforming under Lorentz transformations as proper vector field, in this case the action becomes

$$S = \pm \int d\tau \sqrt{(B \cdot \dot{X})^2 - (B \cdot B) (\dot{X} \cdot \dot{X})}, \quad (33)$$

and this is Lorentz invariant.

Now this system has the primary constraints

$$\Phi_1 = P_M B^M = 0, \quad (34)$$

$$\Phi_2 = P_M B^M + B_M B^M = 0. \quad (35)$$

In consequence the total Hamiltonian is

$$H_T = \lambda_1 \Phi_1 + \lambda_2 \Phi_2. \quad (36)$$

Note that

$$\{P_M B^M, P_N B^N + B_N B^N\} = 2(P_N P_M - B_N B_M) \partial^M B^L. \quad (37)$$

From which it immediately follows that we will get more constraints and these depend on the form of B^M .

An interesting case corresponds to assume that $B^M = X^M$, then the action will be

$$S = \pm \int d\tau \sqrt{(X \cdot \dot{X})^2 - (X \cdot X) (\dot{X} \cdot \dot{X})}, \quad (38)$$

with primary constraints

$$\Phi_1 = P_M X^M = 0, \quad (39)$$

$$\Phi_2 = P_M X^M + X_M X^M = 0, \quad (40)$$

furthermore

$$\{\Phi_1, \Phi_2\} = 2\Phi_3 \quad \Phi_3 = (P_M P^M - X_M X^M). \quad (41)$$

Then, using the Dirac's method, we must satisfy that

$$\Phi_3 = (P_M P^M - X_M X^M) = 0. \quad (42)$$

Now, these constraints satisfy the algebra

$$\{\Phi_1, \Phi_2\} = 2\Phi_3, \quad \{\Phi_1, \Phi_3\} = 2\Phi_2 \quad \{\Phi_2, \Phi_3\} = 8\Phi_1, \quad (43)$$

then, it follows that are first class constraints and there are no more constraints. Thus it appears that the extended Hamiltonian is

$$H_E = \lambda_1 (P_M P^M + X_M X^M) + \lambda_2 P_M X^M + \lambda_3 (P_M P^M - X_M X^M) \quad (44)$$

On the other hand, by defining

$$\phi_1 = \frac{1}{2} P_M P^M, \quad \phi_2 = P_M X^M, \quad \phi_3 = \frac{1}{2} X_M X^M, \quad (45)$$

$$\gamma_1 = \frac{\lambda_1 + \lambda_2}{2}, \quad \gamma_2 = \lambda_2, \quad \gamma_3 = \frac{\lambda_1 - \lambda_2}{2}, \quad (46)$$

we obtain

$$H_E = \gamma_1 \phi_1 + \gamma_2 \phi_2 + \gamma_3 \phi_3. \quad (47)$$

This is the Hamiltonian of the two time physics [9, 14]. We know that the Lagrangian of the two time physics has as a local symmetry the group $Sp(2)$ and as global symmetry the conformal group [14]. Then, the Hamiltonian action of this system has more symmetries than the original action (2).

Let us observe, finally, that the two time physics contains different systems when we use only a temporal coordinate and acts like a model that unifies the dynamics of different systems [9]. In particular, it contains the relativistic free particle. In consequence, by imposing the Lorentz invariance to the Lagrangian (32) we obtain a system that unifies different models to the level of point particle [9].

Finally, it should be emphasized that by consistency, this system requires two temporal coordinates and the signature must be of the form $\text{sig}(\eta) = (-, -, +, \dots, +)$. Thus, to make sense of this system is required that the signature had a transition from $\text{sig}(\eta) = (-, +, +, \dots, +)$ to $\text{sig}(\eta) = (-, -, +, \dots, +)$. It is interesting to mention that recently was proposed materials with this kind of characteristics [15].

6 Poincaré symmetry and relativistic string

The action (38) is Lorentz invariant, but not invariant under the Poincaré group. Now, to write down an explicit Poincaré action we must choose B^M in such way that it be invariant under translations. Note that, if we take $B^M = \frac{\partial C^M}{\partial \tau}$, the Lagrangian (32) is invariant under Poincaré. However, not all the equations of motion will be independent and corresponding system must have constraints. In this case the Hamiltonian analysis is quite involved. Another case, corresponds to take $C^M = X^M$, here the Lagrangian (32) is invariant under Poincaré, but vanishes.

Another way to recover the Poincaré invariance is to consider that the action (2) was built taking as starting point a dispersion relation obtained in Field Theory, i.e., the action was established from a simplification of Field theory to a point particle. It must be stressed that the action for the two time physics was obtained in the same way [10]. Then, to recover de Poincaré invariance we will take the inverse path, i.e., we will transform our particle model into a Field Theory. In fact, assuming that the coordinates depend on an extra parameter σ , i.e. $X^M = X^M(\tau, \sigma)$, it is equivalent to suppose that the particles are not points and instead are linear extended objects. In that case we can take $B^M = T \frac{\partial X^M}{\partial \sigma}$, where T is a constant, and the Lagrangian (32) will be invariant under Poincaré transformations.

We can use the expression $B^M = T \frac{\partial X^M}{\partial \sigma}$, in the constraints (39)-(40) and results

$$\phi_1 = P_M \frac{\partial X^M}{\partial \sigma} = 0, \quad (48)$$

$$\phi_2 = (P_M)^2 + T^2 \frac{\partial X^M}{\partial \sigma} \frac{\partial X_M}{\partial \sigma} = 0. \quad (49)$$

The equations (48) and (49) are the constraints of the Hamiltonian action of the relativistic string [16]. For instance, using $B^M = T \frac{\partial X^M}{\partial \sigma}$ in the Lagrangian (32) we get

$$L = T \sqrt{\left(\frac{\partial X^M}{\partial \sigma} \frac{\partial X_M}{\partial \tau}\right)^2 - \left(\frac{\partial X^N}{\partial \sigma} \frac{\partial X_N}{\partial \sigma}\right) \left(\frac{\partial X^M}{\partial \tau} \frac{\partial X_M}{\partial \tau}\right)}. \quad (50)$$

From this expression, the action takes the form

$$S = T \int d\tau d\sigma \sqrt{\left(\frac{\partial X^M}{\partial \sigma} \frac{\partial X_M}{\partial \tau}\right)^2 - \left(\frac{\partial X^N}{\partial \sigma} \frac{\partial X_N}{\partial \sigma}\right) \left(\frac{\partial X^M}{\partial \tau} \frac{\partial X_M}{\partial \tau}\right)}. \quad (51)$$

and this is the action of Nambu-Goto relativistic string [16].

In this way, by imposing the Lorentz and the Poincaré symmetries to the Lagrangian (32) we get the action of a bosonic string. It should be mentioned that to make Poincaré invariant the Snyder space [11], Yang proposed an extra dimension [12]. In our case we use a similar process, introducing the coordinate σ and we reestablish the Lorentz and Poincaré symmetries.

Furthermore, using the Nambu-Goto (51) we can also recover the action (38). In fact, in order to put this back, we shall use the expression

$$\frac{\partial X^M}{\partial \sigma} = \alpha X^M \quad (52)$$

then

$$X^M(\tau, \sigma) = e^{\alpha\sigma} u^M(\tau). \quad (53)$$

Using this result in (51), we get

$$S = \beta \int d\tau \sqrt{(u \cdot \dot{u})^2 - (u \cdot u) (\dot{u} \cdot \dot{u})}, \quad (54)$$

with $\beta = T|\alpha| \int d\sigma e^{2\alpha\sigma}$. This action is equivalent to (38).

7 Summary

In this work was analyzed several properties of the particle with Lorentz symmetry violation recently proposed by Kostelecký. We introduced an alternative action for this system, that can be interpreted as a particle in a

curved space, where the metric depends on the Lagrange multipliers. In addition, was shown that there exist a limit where this system has more local symmetries than usual relativistic particle. In this limit we saw that there were several forms to reestablish the Lorentz symmetry. In particular, for one of these forms we obtain a relationship with the two time physics. Finally, by recovering the Poincaré symmetry the action of the relativist string was obtained.

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