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Is it possible to relate MOND with Hořava Gravity?

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Abstract

In this work we present a scalar field theory invariant under space-time anisotropic transformations with a dynamic exponent z. It is shown that this theory possess symmetries similar to Hořava gravity and that in the limit $z = 0$ the equations of motion of the non-relativistic MOND theory are obtained. This result allow us to conjecture the existence of a Hořava type gravity that in the limit $z = 0$ is consistent with MOND.

Keyswords: Gravity; Mond; Horava Gravity
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1 Introduction

In recent years, various modifications have been proposed to general relativity. One of the most notorious at a phenomenological level, is the relativistic version of the so called Modified Newtonian Dynamics (MOND) [1]. Notoriously, without making use of dark matter, MOND successfully explains the

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anomalous dynamics of different astrophysical objects. For example, it explains the rotation curves of different galaxies and the Tully-Fisher relation [2]. Besides, MOND has an explanation for the so called Pioneer anomaly [3]. We also have to mention that, MOND presents problems to predict the cluster's of galaxies dynamics, in this case however, the baryonic mass has not been measured with certainty [4].

MOND's starting point is to assume that for small accelerations, about $a_0 \approx 10^{-8} cm/s^2$, Newton's second law takes on the form [1]

$$\mu\left(\frac{|\vec{a}|}{a_0}\right) m\vec{a} = \vec{F}, \tag{1}$$

with $\mu(u)$ defined as a function that satisfies

$$\mu(u) = \begin{cases} 1 & \text{if} \quad u >> 1, \\ u & \text{if} \quad u << 1. \end{cases} \tag{2}$$

The Newtonian regime is obtained if $u >> 1$, meanwhile the MOND's regime is obtained if $u << 1$. In spite of is phenomenological success, MOND has problems at a theoretical level, for example, the energy for one particle is not conserved although it is conserved in several modified versions of the theory [5].

In the non-relativistic gravitational field regime, MOND is consistent with [6]

$$\vec{\nabla} \cdot \left(\mu\left(\frac{|\vec{\nabla}\phi|}{a_0}\right)\vec{\nabla}\phi\right) = 4\pi G\rho, \tag{3}$$

that in the MOND limit takes on the form

$$\vec{\nabla} \cdot \left(\frac{|\vec{\nabla}\phi|}{a_0}\vec{\nabla}\phi\right) = 4\pi G\rho. \tag{4}$$

This model gives a non-relativistic gravitational field description and does not present problems with conserved quantities. However, using this model it is not possible to attack relativistic problems, such as gravitational lenses or cosmological problems. To analyse these problems is necessary a relativistic

MOND. Though there are several relativistic versions of MOND theory, the most complete is the so called $TeVeS$ theory [7], see also [8]. This version is compatible with Eq. (4) in the non-relativistic limit and has been successful in explaining several phenomenological facts. It is worthy to mention that, with the present WMAP measurements, this theory cannot be discarded [9, 10]. With finer WMAP experimental measurements, this theory may be confirmed, modify or even discarded.

However, at a theoretical level, $TeVeS$ still has some inconsistencies, for example, it has some dynamical problems as instabilities may appear [11, 12]. Therefore cannot be considered as a finished theory. In fact, modifications to this theory has been recently proposed [11] and also a new relativistic version of MOND has been recently proposed [13].

In the other hand, Hořava has recently proposed a modified version of general relativity that in principle is renormalizable and free of ghosts [14]. This gravity assumes that space-time is compatible with the anisotropic transformations

$$t \to b^z t, \qquad \vec{x} \to b\vec{x}, \tag{5}$$

where z is a dynamic exponent. As a consequence of the transformation given in Eq. (5), the usual dispersion relation is substituted by

$$P_0^2 - \tilde{G}\left(\vec{P}^2\right)^z = 0, \qquad \tilde{G} = \text{constant}. \tag{6}$$

A remarkable point about this dispersion relation is that, it is not obtained from a geodesic equation [15, 16, 17, 18]. Hořava gravity is not compatible with Lorentz's transformations neither invariant under all the diffeomorphisms, nevertheless for long distances the usual relativity theory is regained. This theory has some dynamical problems [19, 20, 21], and cannot be considered as complete. In fact, some recent proposals have been made in order to improve it [22].

In this work, we will show a scalar field model compatible with the transformations stated in Eq. (5) whose dynamics in the limit $z = 0$ reduces to MOND Eq. (4). It is shown that this theory possess Weyl's symmetries similar to Hořava gravity. This allow us to conjecture that an anisotropic gravity theory in space-time could exist such that, at $z = 0$ reduces to a MOND type gravity; at $z = 1$ standard gravity is regained and at $z = 3$ we regain a Hořava type gravity compatible with quantum mechanics. It worthy to mention that, the anisotropic transformations in Eq. (5) are of importance for the AdS/CFT non-relativistic duality by means of which efforts are made in order to relate condensed matter phenomena with string theory [23, 24]. This would make possible the existence of AdS/CFT duality with a MOND type theory.

This manuscript is organized as follows: In section 2 we present our system and its equations of motion are studied. Section 3 is devoted to study the conserved quantities the system, in Section 4 we study the algebra of the conserved quantities and in Section 5 we present a summary of our results.

2 Action

Consider the Action invariant under the transformation Eq. (5)

$$
\begin{aligned}
S &= \int dx^d dt \left[a \left(\frac{\partial \phi(\vec{x},t)}{\partial t} \right)^{\frac{z+d}{z}} + \gamma \left(\frac{\partial \phi(\vec{x},t)}{\partial x^i} \frac{\partial \phi(\vec{x},t)}{\partial x^i} \right)^{\frac{z+d}{2}} \right] \\
&= \int dx^d dt \left[a \left(\frac{\partial \phi(\vec{x},t)}{\partial t} \right)^{\frac{z+d}{z}} + \gamma \left(\vec{\nabla}\phi \cdot \vec{\nabla}\phi \right)^{\frac{z+d}{2}} \right]. \qquad (7)
\end{aligned}
$$

Note that, if we define the non-zero elements of the metric $g_{\mu\nu}$ as $g_{00} = 1, g_{ij} = \delta_{ij}$, we have $g = \det g_{\mu\nu}$ and therefore the Action Eq. (7) can be written as

$$
S = \int dx^d dt \sqrt{g} \left[a \left(g^{00} \frac{\partial \phi(\vec{x},t)}{\partial t} \frac{\partial \phi(\vec{x},t)}{\partial t} \right)^{\frac{z+d}{2z}} + \gamma \left(g^{ij} \frac{\partial \phi(\vec{x},t)}{\partial x^i} \frac{\partial \phi(\vec{x},t)}{\partial x^j} \right)^{\frac{z+d}{2}} \right], \quad (8)
$$

this expression is invariant under Weyl's anisotropic transformations

$$
g_{00} \to \Omega^{2z}(\vec{x},t) g_{00}, \qquad g_{ij} \to g_{ij} \Omega^2(\vec{x},t). \qquad (9)
$$

This kind of symmetry is similar to the one present at Hořava gravity [14]. The equation of motion for Eq. (7) is

$$a\left(\frac{z+d}{z}\right)\frac{\partial}{\partial t}\left(\frac{\partial\phi}{\partial t}\right)^{\frac{d}{z}}+\gamma\left(z+d\right)\frac{\partial}{\partial x_i}\left(\left(\frac{\partial\phi}{\partial x^j}\frac{\partial}{\partial x^j}\right)^{\frac{z+d-2}{2}}\frac{\partial\phi}{\partial x^i}\right)=0,$$

that can be written as

$$a\left(\frac{z+d}{z}\right)\frac{\partial}{\partial t}\left(\frac{\partial\phi}{\partial t}\right)^{\frac{d}{z}}+\gamma\left(z+d\right)\vec{\nabla}\cdot\left(|\vec{\nabla}\phi|^{d+z-2}\vec{\nabla}\phi\right)=0. \tag{10}$$

If a source ρ is consider, the Action Eq. (7) is now given by

$$S=\int dx^d dt\left[a\left(\frac{\partial\phi}{\partial t}\right)^{\frac{z+d}{z}}+\gamma\left(\vec{\nabla}\phi\cdot\vec{\nabla}\phi\right)^{\frac{z+d}{2}}+\phi\rho\right]. \tag{11}$$

If under scaling the source is transformed as $\rho\rightarrow\Omega^{-(z+d)}\rho$, then Eq. (11) is invariant under Weyl's symmetry, Eq. (9). The equation of motion given by the Action Eq. (11) is

$$a\left(\frac{z+d}{z}\right)\frac{\partial}{\partial t}\left(\frac{\partial\phi}{\partial t}\right)^{\frac{d}{z}}+\gamma\left(z+d\right)\vec{\nabla}\cdot\left(|\vec{\nabla}\phi|^{d+z-2}\vec{\nabla}\phi\right)=-\rho. \tag{12}$$

It can be noticed that, at the limit $z\rightarrow 0$ the first term of the Action in Eq. (11) is constant and we can obtain the effective Action as

$$S=\int dx^d\left[\gamma\left(\vec{\nabla}\phi\cdot\vec{\nabla}\phi\right)^{\frac{d}{2}}+\phi\rho\right], \tag{13}$$

whose equation of motion is

$$\gamma d\vec{\nabla}\cdot\left(|\vec{\nabla}\phi|^{d-2}\vec{\nabla}\phi\right)=-\rho. \tag{14}$$

If $d=3$ and $\gamma=-1/(12\pi Ga_0)$ we obtain the MOND non-relativistic equation of motion Eq. (4), [6].

Therefore the system under study contains the non-relativistic MOND's theory and coincides with Hořava gravity symmetries. This fact, make it possible to conjecture the existence of a Hořava type gravity that, in $z=0$ limit reduces to MOND. Note that, as the Hořava gravity must be valid in the quantum regime, the fundamental constant is Planck's mass M_P. This constant seems to be non-related with MOND's fundamental constant a_0. However, a_0 can be written as $a_0 \approx m_N c \left(6M_P^3 t_p\right)^{-1}$, where m_N is the proton mass. It is possible that in this conjectured gravity this type of relations could arise in a natural way. It can be noticed that, in this relation we have a collection of apparently dissimilar quantities, however, by means of the Holographic principle, this type of relations appear [25].

3 Noether's theorem

In this section, we will find out the conserved quantities of the Action Eq. (7). First, note that the canonical momentum is given by

$$\Pi = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = a\frac{z+d}{z}\left(\dot{\phi}\right)^{\frac{d}{z}}, \tag{15}$$

therefore, the equation of motion can be written as

$$\frac{\partial \Pi}{\partial t} + \gamma(z+d)\vec{\nabla}\cdot\left(|\vec{\nabla}\phi|^{d+z-2}\vec{\nabla}\phi\right) = -\rho. \tag{16}$$

Considering Noether's theorem, we know that the temporal part of

$$\int d^d J_\mu = \int dx^d \left(\frac{\partial \mathcal{L}}{\partial(\partial^\mu \phi)}\frac{\partial \phi}{\partial x^\nu} - g_{\mu\nu}\mathcal{L}\right)\delta x^\nu \tag{17}$$

is conserved. Taking into account that the Action Eq. (7) is invariant under temporal translations, we can conclude that the Hamiltonian is conserved

$$H = \int dx^d \mathcal{H} = \int dx^d \left(\frac{ad}{z}\left(\frac{z}{a(z+d)}\right)^{\frac{d+z}{d}}\Pi^{\frac{z+d}{d}} - \gamma|\vec{\nabla}\phi|^{z+d}\right). \tag{18}$$

Besides, the momentum is conserved

$$P_i = \int dx^d p_i = \int dx^d \Pi \frac{\partial \phi}{\partial x^i} = \int dx^d a\frac{z+d}{z}\left(\dot{\phi}\right)^{\frac{d}{z}}\frac{\partial \phi}{\partial x^i} \tag{19}$$

as well as the angular momentum

$$L_i = -\int dx^d \Pi \epsilon_{ijk} x_j \frac{\partial \phi}{\partial x^k} = -\int dx^d a\frac{z+d}{z}\left(\dot{\phi}\right)^{\frac{d}{z}}\epsilon_{ijk}x_j\frac{\partial \phi}{\partial x^k}. \tag{20}$$

The scaling generator

$$D = \int dx^d \left(zt\mathcal{H} + p_i x^i \right) \tag{21}$$

is also conserved. This quantities form the algebra

$$\begin{aligned}
\{H, P_i\} &= 0, && (22)\\
\{H, L_{kl}\} &= 0, && (23)\\
\{H, D\} &= zH, && (24)\\
\{D, P_i\} &= -P_i, && (25)\\
\{D, L_i\} &= 0, && (26)\\
\{P_i, P_j\} &= 0, && (27)\\
\{P_i, L_j\} &= \epsilon_{ijm} P_m, && (28)\\
\{L_i, L_j\} &= \epsilon_{ijk} L_k. && (29)
\end{aligned}$$

This type of algebraic relations are characteristic of anisotropic scale invariant systems with dynamic exponent z.

4 Summary

In this work we have presented a scalar field theory invariant under space-time anisotropic transformations with a dynamic exponent z. It is shown that this theory possess symmetries similar to Hořava gravity, in particular Weyl's symmetries. Also, it is shown that in the limit $z = 0$ the equations of motion of the non-relativistic MOND theory are obtained. This result make it possible to conjecture the existence of a Hořava type gravity that in the limit $z = 0$ is consistent with MOND. Also, conserved quantities and their algebraic relations have been studied.

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