Conformal Anisotropic Mechanics

Juan M. Romero[*]

Departamento de Matemáticas Aplicadas y Sistemas
Universidad Autónoma Metropolitana-Cuajimalpa, México 11950 DF, México

Vladimir Cuesta,[†] J. Antonio Garcia,[‡] and J. David Vergara[§]

Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México,
Apartado Postal 70-543, México 04510 DF, México

(Dated: October 30, 2018)

In this paper we implement scale anisotropic transformations in the space-time in classical mechanics. The resulting system is consistent with the dispersion relation of gravity at a Lifshitz point recently considered in [9]. Also, we show that our model is a generalization of the conformal mechanics of Alfaro, Fubini and Furlan. For arbitrary z we construct the dynamical symmetries that correspond to the Schroedinger algebra. Furthermore, we obtain the Boltzman distribution for a gas of free particles compatible with anisotropic scaling transformations and compare our result with the corresponding thermodynamics of the recent anisotropic black branes proposed in [8].

PACS numbers: Valid PACS appear here

I. INTRODUCTION

Scaling space-time symmetries are very useful tools to study non linear physical systems and critical phenomena [1, 2]. At the level of classical mechanics an interesting example is the conformal mechanics [3] that is relevant for several physical systems from molecular physics to black holes [4, 5]. Recently a new class of anisotropic scaling transformations has been considered in the context of critical phenomena [6], string theory [7, 8, 16] and quantum gravity [9].

$$t \to b^z t, \qquad \vec{x} \to b\vec{x}. \tag{1}$$

where z play the role of a dynamical critical exponent. In particular the result of Horava [9] is quite interesting since it produces a theory that reduces to general relativity at large scales and may provide a candidate for a UV completion of general relativity. In principle this model for Gravity is renormalizable in the sense that the effective coupling constant is dimensionless in the UV limit. A fundamental property of these gravity models is that the scaling laws (1) are not compatible with the usual relativity. In the IR these systems approach the usual General Relativity with local Lorentz invariance but in the UV the formulation admit generalized dispersion relations of the form

$$P_0^2 - G(\vec{P}^2)^z = 0, \qquad G = \text{const.} \tag{2}$$

Where z plays the role of a dynamical exponent. As the dispersion relations used by these models are quadratic in

P_0 while the spatial momentum scale as z the models are in principle renormalizable by power counting arguments at least for $z = 3$. It is claimed also that these models are ghost free. On the other hand due to the non relativistic character of the Horava gravity, this theory must be considered as an effective version of a more fundamental theory, compatible with the symmetries that we observe in our universe. The consideration of these models could be useful to understand the quantum effects of gravity.

In order to gain some understanding of the content of generalized dispersion relations of the form (2) we will construct a reparametrization invariant classical mechanics compatible with the scaling laws (1). In particular this implementation of the scaling transformations (1) will lead to dispersion relations of the form (2). Another interesting characteristic of our implementation is that it contains interesting systems in the limits $z = 2$ where the conformal mechanics of Fubini, et al [3] is recovered, $z = 1$ where we recover the standard relativistic particle and $z \to \infty$ where we obtain the Euclidian relativistic particle.

We also study the symmetries of the anisotropic mechanics and compare our results with the Schroedinger algebra for any value of z. Finally, by analyzing the thermodynamic properties of a free gas compatible with the scaling transformation (1) we will obtain the same thermodynamic relations for black branes recently proposed in [8].

The organization of the paper is as follows: Section 2 concerns with the action of the system. In Sec. 3 we study the canonical formalism of the system and his gauge symmetries. Sec. 4 focuses on the equations of motion. The global symmetries and the Schroedinger algebra are analyzed in section 5. In Sec. 6 we consider the thermodynamic properties of the system and we conclude in section 7.

[*]Electronic address: jromero@correo.cua.uam.mx
[†]Electronic address: vladimir.cuesta@nucleares.unam.mx
[‡]Electronic address: garcia@nucleares.unam.mx
[§]Electronic address: vergara@nucleares.unam.mx

II. THE ACTION PRINCIPLE

Scale transformation can be realized in non-relativistic theories in different ways. From one hand we can start from an action in curved space where some of its isometries are the scale transformations. Another way to implement scale transformations is to start from an action in flat space but constructed in such way that the invariance under scale transformations is manifest. In this work will take this second point of view. Let us consider a $d+1-$dimensional space-time, with an action principle given by

$$S = \int d\tau L = \int d\tau \left(\frac{m}{2} \frac{\left(\dot{x}^2\right)^{\frac{z}{2(z-1)}}}{\left(\dot{t}\right)^{\frac{1}{z-1}}} - V\left(\vec{x}\right)\dot{t} \right), \quad (3)$$

$$\dot{t} = \frac{dt}{d\tau}, \quad \dot{x}_i = \frac{dx_i}{d\tau}, \quad \dot{x}^2 = \dot{x}_i \dot{x}^i, \quad i = 1, 2, \cdots, d.$$

This action is invariant under reparametrizations of τ, i.e. under the transformations $\tau \to \tau = \tau(f)$, if we require that t and the x_i are scalars under reparametrizations. Furthermore, if we consider that the potential satisfies $V(b\vec{x}) = b^{-z}V(\vec{x})$, for example,

$$V(\vec{x}) = \frac{a}{|\vec{x}|^z}, \qquad a = \text{cte} \quad (4)$$

the action (3) is invariant under the scaling laws Eq.(1). Note that, when $z = 2$ and $\tau = t$ we obtain the action for a non-relativistic particle, furthermore taking into account (4), we obtain

$$S = \int dt \left(\frac{m}{2} \dot{x}^2 - \frac{a}{|\vec{x}|^2} \right), \quad (5)$$

that corresponds to the action of the conformal mechanics [3]. In the case of arbitrary z and for a gauge condition $\tau = t$ we get

$$S = \int dt \left(\frac{m}{2} \left(\dot{x}^2\right)^{\frac{z}{2(z-1)}} - \frac{a}{|\vec{x}|^z} \right), \quad (6)$$

that again is invariant under the scaling laws (1). In this sense Eq.(3) represents a generalized conformal mechanics.

Another interesting property of the action (3) appears in the limit $z \to \infty$. In this case we obtain

$$S = \int d\tau \left(\frac{m}{2} \sqrt{\dot{x}^2} - V\left(\vec{x}\right)\dot{t} \right), \quad (7)$$

this action is invariant under reparametrizations, and leaving $V(\vec{x}) = 0$ we obtain the action for an Euclidean relativistic particle

$$S = \int d\tau \frac{m}{2} \sqrt{\dot{x}^2}. \quad (8)$$

Another interesting limit is in the case $z = 1$. Even though this limit is not well defined for the action (3)

it can be implemented using an equivalent form of the action (3)

$$S = \int d\tau \frac{m}{2(z-1)} \left(\frac{\left(\dot{x}^2\right)^{\frac{z}{2}}}{e^{z-2}\dot{t}} + (z-2)e \right). \quad (9)$$

Here, we have introduced an einbein e in a similar way as the standard trick to remove the square root in the action of the free relativistic particle. Using this action (9) we can now take the limit $z \to 1$ if at the same time we take the limit $m \to 0$. Assuming that $m \to 0$ at the same rate as $z-1$ in such way that $\frac{m}{z-1} \to 1$ we obtain an action that is very similar to the action of the massless relativistic free particle. We conclude the the limit $z \to 1$ corresponds to the massless relativistic particle that is invariant under the full group of relativistic conformal transformations.

III. CANONICAL FORMALISM

Let us consider the canonical formalism of the system (3). The canonical momenta are

$$p_i = \frac{\partial L}{\partial \dot{x}^i} = \frac{mz}{2(z-1)} \frac{\left(\dot{x}^2\right)^{\frac{2-z}{2(z-1)}}}{\left(\dot{t}\right)^{\frac{1}{z-1}}} \dot{x}_i, \quad (10)$$

$$p_t = \frac{\partial L}{\partial \dot{t}} = -\left(\frac{m}{2(z-1)} \frac{\left(\dot{x}^2\right)^{\frac{z}{2(z-1)}}}{\left(\dot{t}\right)^{\frac{z}{z-1}}} + V(\vec{x}) \right) = -H. \quad (11)$$

and as expected (by the reparametrization invariance) the canonical Hamiltonian vanishes

$$H_c = p_i \dot{x}^i + p_t \dot{t} - L = 0. \quad (12)$$

From (10) we get

$$p^2 = p_i p^i = \left(\frac{mz}{2(z-1)} \frac{\left(\dot{x}^2\right)^{\frac{1}{2(z-1)}}}{\left(\dot{t}\right)^{\frac{1}{z-1}}} \right)^2, \quad (13)$$

that implies in turn

$$\phi = p_t + \frac{m}{2(z-1)} \left(\frac{mz}{2(z-1)} \right)^{-z} \left(p^2\right)^{\frac{z}{2}} + V(\vec{x}) \approx 0. \quad (14)$$

If we enforce the constraint, and considering the case when $V(\vec{x}) = 0$ we obtain

$$p_t = -\frac{m}{2(z-1)} \left(\frac{mz}{2(z-1)} \right)^{-z} \left(p^2\right)^{\frac{z}{2}}, \quad (15)$$

which can be rewritten in the form

$$(p_t)^2 - G\left(p^2\right)^z = 0, \qquad G = \frac{1}{z^{2z}} \left(\frac{m}{2(z-1)} \right)^{2(1-z)} \quad (16)$$

In this way, we get a dispersion relation that is very similar to the Horava's gravity model Eq. (2).

Notice that by taking the limits $z \to 1$ and $m \to 0$ in (14) we get the constraint

$$\phi_1 = p_t + (p^2)^{\frac{z}{2}} + V(\overrightarrow{x}), \qquad (17)$$

that is consistent with the canonical formalism of the action (9) and corresponds to a massless relativistic particle with only positive energy.

The system has one first class constraint (14) and following the standard Dirac's method [10], the "wave equation"

$$\hat{\phi}|\psi> = \left(\hat{p}_t + \sqrt{G}\left(\hat{p}^2\right)^{\frac{z}{2}} + V(\vec{x})\right)|\psi> = 0. \qquad (18)$$

must be implemented on the Hilbert space at quantum level to obtain the physical states. In other words, the "Schroedinger equation" corresponding to the anisotropic mechanics (3) is

$$-i\hbar\frac{\partial\psi}{\partial t} = \left(\sqrt{G}\left(-\hbar^2\nabla^2\right)^{\frac{z}{2}} + V(\vec{x})\right)\psi. \qquad (19)$$

Gauge of freedom

Because of the action (3) is invariant under reparametrizations, there is a local symmetry. The extended canonical action for the system is [11]

$$S = \int d\tau \left(\dot{t}p_t + \dot{x}^i p_i - \lambda\phi\right) \qquad (20)$$

with λ a Lagrange multiplier.

If we require that ϵ is a function of τ the extended action is invariant under the gauge transformations [11]

$$
\begin{aligned}
\delta x_{i\epsilon} &= \{x_i, \epsilon\phi\} = \epsilon\sqrt{G}z(p^2)^{\frac{z-2}{2}}p_i,\\
\delta p_{i\epsilon} &= \{p_i, \epsilon\phi\} = -\epsilon\frac{\partial V}{\partial x_i},\\
\delta t_\epsilon &= \epsilon\{t, \phi\} = \epsilon,\\
\delta p_{t\epsilon} &= 0,\\
\delta\lambda_\epsilon &= \frac{\partial\epsilon}{\partial\tau}.
\end{aligned}
$$

Here $\{,\}$ are the Poisson brackets. The above transformations, are of the usual type for the parametrized particle. However, the rule for the transformation corresponding to the spatial coordinates seems to change drastically. But if we use the definition of the momenta (10) we will get exactly the usual transformation $\delta x_{i\epsilon} = \epsilon\frac{dx^i}{dt}$ recovering the full diffeomorphism transformations associated with the reparametrization invariance. This result can be constrasted with the same result obtained by Horava's in his gravity model [9].

In this way we saw that the action Eq.(3), has very similar properties to the Horava's gravity namely the same scaling properties and the same invariance under reparametrizations. In this sense, we see that our toy model has the same transformation properties than the Horava's gravity just as the relativistic particle has the same transformation properties as General Relativity [12].

IV. EQUATIONS OF MOTION

From the variation of the action (3), and taking into account the definitions of the momenta p_t and p_i, it follows that

$$
\begin{aligned}
\delta S &= \int d\tau \left(\frac{d}{d\tau}\left(p_i\delta x_i + p_t\delta t\right)\right. \qquad (21)\\
&\left. -\delta x_i\left(\frac{dp_i}{d\tau} + \frac{\partial V}{\partial x_i}\right) + \delta t\frac{dp_t}{d\tau}\right),
\end{aligned}
$$

where the Euler-Lagrange equations are

$$\frac{dp_i}{d\tau} + \frac{\partial V}{\partial x_i} = 0, \qquad (22)$$

$$\frac{d}{d\tau}p_t = 0. \qquad (23)$$

Notice that for $z = 2$ and $\tau = t$ the equation (22) we obtain the usual Newton's second law, and that (23) correspond to the energy conservation.

In order to simplify some computations we will use the gauge condition $\tau = t$. This is a good gauge condition in the sense that $\{\phi, \tau - t\} \neq 0$. Under the above assumption and using the definition p_i we get the equations of motion

$$\frac{d}{dt}\left(\left(\frac{mz}{2(z-1)}\right)(\dot{x}^2)^{\frac{2-z}{2(z-1)}}\dot{x}^i\right) + \frac{\partial V}{\partial x_i} = 0, \qquad (24)$$

We thus get

$$
\begin{aligned}
g_{ij}\ddot{x}_j &= -\left(\dot{x}^2\right)^{\frac{z-2}{2(z-1)}}\frac{2(z-1)}{mz}\frac{\partial V}{\partial x_i},\\
g_{ij} &= \left(\delta_{ij} + \frac{2-z}{z-1}\frac{\dot{x}_i\dot{x}_j}{\dot{x}^2}\right). \qquad (25)
\end{aligned}
$$

The matrix g_{ij} can be considered as a metric that depends on the velocities. We see that this metric is homogeneous of degree zero in the velocities and in consequence is a Finsler type metric [13]. This metric has three critical points:

$$z = 1, \qquad z = 2, \qquad z = \infty. \qquad (26)$$

For $z \neq \infty$, we obtain the inverse metric

$$g^{ij} = \left(\delta^{ij} - (2-z)\frac{\dot{x}^i\dot{x}^j}{\dot{x}^2}\right), \qquad (27)$$

it follows that

$$\ddot{x}_i = \frac{2(1-z)}{mz}\left(\dot{x}^2\right)^{\frac{z-2}{2(z-1)}}\left(\delta_{ij} + (z-2)\frac{\dot{x}_i\dot{x}_j}{\dot{x}^2}\right)\frac{\partial V}{\partial x_j}. (28)$$

The limit $z = \infty$ is particularly interesting, since in this case the matrix g_{ij} is not invertible. This means that the gauge condition $\tau = t$ is not valid. Because of in this limit we get the action (8) and in this case is not more valid the constraint (14).

V. DYNAMICAL SYMMETRIES OF THE SCHROEDINGER EQUATION FOR ANY z

In the same sense as the Galilei group exhaust all the symmetries of the nonrelativistic free particle in d dimensions the Schroedinger Algebra is the algebra of symmetries of the Schroedinger equation with $V = 0$ and can be considered as the non relativistic limit of the Conformal symmetry algebra. Another interesting Galilean Conformal Algebra can also be constructed as the non relativistic contraction of the full Conformal Relativistic Algebra [14]. The generators of the Schroedinger algebra include temporal translations H, spatial translations P^i, rotations J^{ij}, Galilean boost K^i, dilatation D (where time and space can dilate with different factors), special conformal transformation C, and the mass operator M. The nonzero commutators of this algebra are the Galilean algebra

$$
\begin{aligned}
\{J^{ij}, J^{kl}\} &= (\delta^{ik}J^{jl} + \delta^{jl}J^{ik} - \delta^{il}J^{jk} - \delta^{jk}J^{il}), \\
\{J^{ij}, P^k\} &= (\delta^{ik}P^j - \delta^{jk}P^i), \\
\{J^{ij}, K^k\} &= (\delta^{ik}K^j - \delta^{jk}K^i), \\
\{H, K^i\} &= P^i \\
\{P^i, K^j\} &= -\delta^{ij}M,
\end{aligned} \quad (29)
$$

plus Dilatations

$$
\{D, P^i\} = P^i, \quad \{D, K^i\} = K^i, \quad \{D, H\} = -2H \quad (30)
$$

and Special Conformal transformations

$$
\{D, C\} = -2C, \quad \{H, C\} = D, \quad \{C, P^i\} = K^i. \quad (31)
$$

It is interesting to notice that H, C, D close themselves in an $SL(2, \mathbb{R})$ subalgebra. Thus the Schroedinger algebra is the Galilean algebra plus dilatation and Special Conformal transformations [15]. M is the center of the algebra.

It is also worth noticing that the Schroedinger algebra in d dimensions, $Schr(d)$, can be embedded into the relativistic conformal algebra in $d + 2$ space-time dimensions $O(d + 2, 2)$. This fact is central in the recent literature about the geometric content of the new AdS/NRCFT dualities (for details see [16]).

We are interested in the construction of the explicit generator of the relative Poisson-Lie Schroedinger algebra for any z. These will be the symmetries of the corresponding anisotropic Schroedinger equation (15) associated with the non-relativistic anisotropic classical mechanics (3).

We will denote the generalization of the Schroedinger symmetry algebra for any z as $Schr_z(d)$ in d dimensions. The algebra is given by the Galilean algebra (29) plus

$$
\begin{aligned}
\{D, P^i\} &= P^i, \quad \{D, K^i\} = (1 - z)K^i, \\
\{D, M\} &= (2 - z)M, \quad \{D, H\} = -zH
\end{aligned} \quad (32)
$$

Notice that M is not playing the role of the center anymore unless $z = 2$ and that C, the generator of the Special Conformal transformations is not in the algebra [17]. We are not aware of any explicit realization of this algebra in phase space for arbitrary dynamical exponent z. For $z = 2$ the phase space realization reads

$$
\begin{aligned}
J^{ij} &= x^i p^j - x^j p^i, \quad H = -p_t, \quad P^i = p^i, \\
K^i &= -Mx^i + tp^i \\
C &= t^2 p_t + tx^i p^i - \frac{1}{2}Mx^2, \quad D = 2tp_t + x^i p^i
\end{aligned}
$$

Inspired by this construction and the embedding of the Schroedinger algebra in d dimensions into the full relativistic Conformal algebra in $d + 2$ [16] we will display a set of dynamical symmetries of the Schroedinger equation for any z.

The crux of the argument is to allow M to depend on the magnitude of p^i squared $M(p^2)$. We will make the anzats that M is a homogeneus function of degree α in p^i

$$
\frac{\partial M}{\partial p^i}p^i = 2\alpha M, \qquad M(p^2) = A(p^2)^\alpha, \qquad (33)
$$

where A is a constant. Then taking as our Schroedinger equation (14) with $V(\overrightarrow{x}) = 0$ and for any z we have

$$
S = p_t + H = p_t + \frac{p^2}{2M(p^2)} \qquad (34)
$$

we will ask for the phase space quantities that commutes (in Poisson sense) with the equation (34), i.e.,

$$
\{\mathcal{O}, S\} = 0 \qquad (35)
$$

over $S = 0$. Any such phase space quantity will be a dynamical symmetry of the anisotropic Schroedinger equation (15). α is fixed in terms of the dynamical exponent z by relating our definition (34) with the first class constraint (15)

$$
\alpha = \frac{2 - z}{2}, \qquad A = \frac{1}{2\sqrt{G}} \qquad (36)
$$

J^{ij}, P^i, H are obvious symmetries of the anisotropic Schroedinger equation. To see what are the analogs of D, C and K^i let us start with D as defined for the $z = 2$ case but with an anisotropic reescaling between space and time

$$
D = tp_t + \beta x^i p_i
$$

By taking the commutator with S we have

$$\{D, S\} = p_t + \frac{\beta(1-\alpha)p^2}{M(p^2)}$$

Then

$$\beta = \frac{1}{2(1-\alpha)}$$

Using the same idea with the Special Conformal generator we propose

$$C = t^2 p_t + \gamma t x^i p_i + \sigma M(p^2) x^2$$

where γ, σ are constants to be determined from the requirement (35). The solutions are

$$\gamma = -\frac{1}{(1-\alpha)}, \qquad \sigma = -\frac{2}{(1-\alpha)^2}$$

For the Galilean boost we have

$$K^i = t p^i + \rho M(p^2) x^i$$

and the solution for the constant ρ using the condition (35) is

$$\rho = -\frac{1}{(1-\alpha)}$$

So we have the non trivial dynamical symmetries of the anisotropic Schroedinger equation generated by

$$D = zt p_t + x^i p_i, \tag{37}$$

$$C = \frac{z^2}{4} t^2 p_t + \frac{z}{2} t x^i p_i - \frac{1}{2} M(p^2) x^2, \tag{38}$$

that are dilatation and Special Conformal transformations and d Galilean boosts

$$K^i = \frac{z}{2} t p^i - M(p^2) x^i. \tag{39}$$

A similar result was obtained previously in [6] for a differential representation of the algebra (32) using the concept of fractional derivatives.

Unfortunately the dynamical symmetries given by (37), (38) and (39) plus J^{ij}, P^i, H, M do not close in $Schr_z(d)$ (32). Nevertheless a subalgebra formed by H, C, D indeed close into an $SL(2, \mathbb{R})$ sector of the full algebra,

$$\{D, H\} = -zH, \qquad \{D, C\} = -zC, \qquad \{H, C\} = \frac{z}{2} D,$$

To these generator we can add the angular momentum J^{ij} that commutes with them. From the other hand the anisotropic system (3) is also invariant under the Poincaré symmetry alone.

VI. THERMODYNAMIC PROPERTIES

In the following we shall sketch some of the thermodynamic properties of the system. We will consider only the case without potential. In this case the energy of our model is

$$H = \sqrt{G} p^z. \tag{40}$$

Let us denote by $\mathbf{V}$ the volume and $\beta = \frac{1}{kT}$, with T the temperature and k the Boltzmann constant, the canonical partition function for the system in a space of d dimensions is

$$\begin{aligned}
Z &= \frac{\mathbf{V}}{\hbar^N} \int d\vec{p} e^{-\beta\sqrt{G}p^z} = \frac{\mathbf{V}}{\hbar^N} \int d\Omega_d dp p^{d-1} e^{-\beta\sqrt{G}p^z} \\
&= \frac{\mathbf{V}\Omega_d}{\hbar^N} \int_0^\infty dp p^{d-1} e^{-\beta\sqrt{G}p^z}. \tag{41}
\end{aligned}$$

Using $u = \beta\sqrt{G}p^z$, and the definition of the Gamma function we obtain

$$Z = \frac{\mathbf{V}\Omega_d}{z\hbar^N \left(\beta\sqrt{G}\right)^{\frac{d}{z}}} \Gamma\left(\frac{d}{z}\right). \tag{42}$$

In this way, if N is the number of particles, the Helmholtz free energy is given by

$$F = -kTN \ln Z = -kTN \ln\left(\frac{\mathbf{V}\Omega_d}{z\hbar^N \left(\beta\sqrt{G}\right)^{\frac{d}{z}}} \Gamma\left(\frac{d}{z}\right)\right) \tag{43}$$

From this expression we may now obtain all the thermodynamic properties of the system, in particular the internal energy is

$$U = NkT^2 \frac{\partial \ln Z}{\partial T} = \frac{d}{z} NkT. \tag{44}$$

where N is the total number of particles in the gas. Substituting the number of particles N in terms of the partition function ($Z = N$ in our normalization) the internal energy can be written as

$$U = \frac{\mathbf{V}\Omega_d \Gamma\left(\frac{d}{z}\right)}{z\hbar^N \left(\sqrt{G}\right)^{\frac{d}{z}}} \frac{d}{z} (kT)^{\frac{d+z}{z}}. \tag{45}$$

Whereas the entropy is

$$S = \int dT \frac{1}{T} \left(\frac{\partial U}{\partial T}\right)_{\mathbf{V}} = \frac{\mathbf{V}\Omega_d \Gamma\left(\frac{d}{z}\right)(k)^{\frac{d+z}{z}}}{z\hbar^N \left(\sqrt{G}\right)^{\frac{d}{z}}} \frac{d}{z} (T)^{\frac{d}{z}} \frac{d+z}{d} \tag{46}$$

We conclude from (45) and (46), that the relationship between energy and entropy in our system is given by

$$U = \frac{STd}{d+z}. \tag{47}$$

This relationship is exactly the obtained in the case of black branes [8].

VII. CONCLUSIONS

In this work we have introduced an action invariant under anisotropic transformations in space and time. This anisotropic mechanical system is consistent with the non-relativistic Horava's dispersion relation. Furthermore, it was shown that for our particle model our system has the same local symmetries. Another interesting point is that our system corresponds to the Conformal Mechanics of [3] for $z = 2$ and is a generalization of this system for arbitrary z. Also our system includes in the limit $z \to \infty$ a Euclidean relativistic particle and in the limit $z \to 1$ a massless relativistic particle. From the equations of motion we show that naturally appears a Finsler type metric that could implies that the Horava's gravity can be related to Finsler geometry. Also, we studied the dynamical symmetries of our model and we found all dynamical symmetries of the anisotropic Schroedinger equation for arbitrary z that correspond to the generators of the Schroedinger algebra $Schr_z(d)$. We show that the full Schroedinger algebra constructed from our generators does not close. However we found that a subalgebra $SL(2, \mathbb{R})$ indeed close. An interesting point is that the explicit realization of the generators is not linear and are analogous to the realization given in [6]. As a final point, we remark that the thermodynamic properties of our model reproduces the same thermodynamic properties of the recent anisotropic black branes.

Acknowledgments

[1] P.C. Hohenberg, B.I. Halperin, *Theory of Dynamic Critical Phenomena,* Rev. Mod. Phys. **49** 435 (1977).

[2] J. L. Cardy, *Scaling And Renormalization In Statistical Physics,* Cambridge, UK: Univ. Pr. (1996).

[3] V. Alfaro, S. Fubini, G. Furlan, *Conformal Invariance in Quantum Mechanics,* Nuovo Cim. A **34**, 569 (1976).

[4] R. Britto-Pacumio, J. Michelson, A. Strominger, A. Volovich, *Lectures on superconformal quantum mechanics and multiblack hole moduli spaces,* Contributed to NATO Advanced Study Institute on Quantum Geometry. Published in Progress in string theory and M-theory, 235-264 (1999), hep-th/9911066.

[5] H. E. Camblong, C. R. Ordonez, *Anomaly in conformal quantum mechanics: From molecular physics to black holes,* Phys. Rev. D **68** 125013 (2003), hep-th/0303166.

[6] M. Henkel, "Phenomenology of local scale invariance: From conformal invariance to dynamical scaling," Nucl. Phys. B **641**, 405 (2002) [arXiv:hep-th/0205256].

[7] J.L.F. Barbon, C.A. Fuertes *On the spectrum of nonrelativistic AdS/CFT,* JHEP 0809:030 (2008), arXiv:0806.3244.

[8] G. Bertoldi, B. A. Burrington, A. Peet, *Thermodynamics of black branes in asymptotically Lifshitz spacetimes,* arXiv:0907.4755.

[9] P. Horava, *Quantum Gravity at a Lifshitz Point,* Phys. Rev. D, **79** 084008 (2009). arXiv:0901.3775.

[10] P. A. M. Dirac, *Lectures on Quantum Mechanics,* Dover, New York, (2001).

[11] M. Henneaux and C. Teitelboim, *Quantization of Gauge System,* Princeton University Press, USA, (1992).

[12] C. Teitelboim, *Quantum Mechanics of the Gravitational Field,* Phys. Rev. D **25** 3159 (1982).

[13] F. Girelli, S. Liberati, L. Sindoni, *Planck-scale modified dispersion relations and Finsler geometry,* Phys. Rev. D **75**, 064015 (2007), gr-qc/0611024.

[14] A. Bagchi and R. Gopakumar, "Galilean Conformal Algebras and AdS/CFT," JHEP **0907**, 037 (2009) [arXiv:0902.1385 [hep-th]].

[15] C. R. Hagen, "Scale and conformal transformations in galilean-covariant field theory," Phys. Rev. D **5**, 377 (1972). U. Niederer, "The maximal kinematical invariance group of the free Schrodinger equation," Helv. Phys. Acta **45** (1972) 802.

[16] D. T. Son, "Toward an AdS/cold atoms correspondence: a geometric realization of the Schroedinger symmetry," Phys. Rev. D **78**, 046003 (2008) [arXiv:0804.3972 [hep-th]].

[17] K. Balasubramanian and J. McGreevy, "Gravity duals for non-relativistic CFTs," Phys. Rev. Lett. **101**, 061601 (2008) [arXiv:0804.4053 [hep-th]].